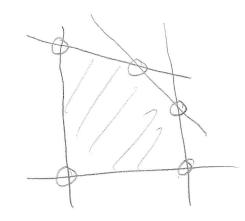
Linear Programming:

The method of linear programming is a procedure we use to optimize (max/min) an *objective* subject to *constraints*. Here is the short version:

- Step 1: Label the two quantities Collect information (in a table)
- Step 2: Give Constraints and Objective.
- Step 3: Sketch the Feasible Region.
- Step 4: Find the corners.
- Step 5: Plug corners into the Objective. Biggest output = max Smallest output = min



PLUG THESE FIVE

POINTS INTO THE OBJECTIVE (THE JHING WE LANT TO OPTIMIZE)

니오: Linear Programming Examples (뚜rơm the lecture pack)

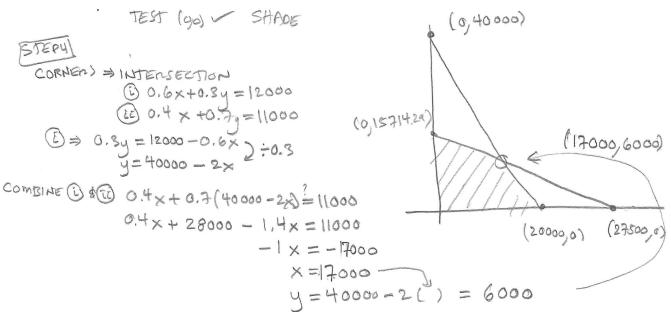
Your company makes two fruit juices. Apple-Cranberry is 60% apple juice and 40% cranberry juice. Cranberry-Apple is 30% apple and 70% cranberry.

You make \$0.40 profit on each gallon of Apple-Cranberry and \$0.50 profit on each gallon of Cranberry-Apple. Your daily Supply is limited to 12,000 gallons of pure apple juice and 11,000 gallons of pure Cranberry juice.

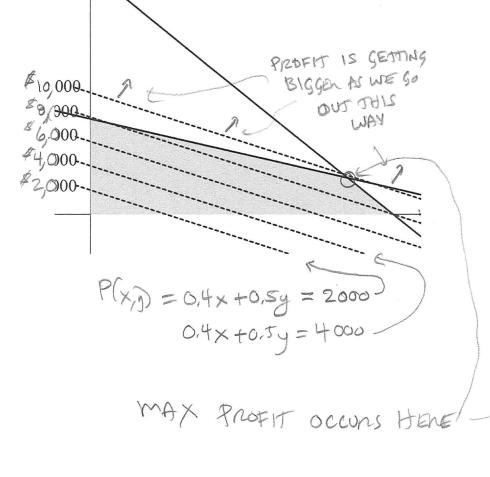
Defermine how much of each you should pro duce daily to maximize your profit.

T	TEP 1: "How much of each "					
	x = "gallons	of Appl	e-Crawbern	4 11		
	y = "gallons	of Crar	n-Apple	0		
	Gallons =	х	У			
	APPLEJUICE	0.6×	0.34			
	CRANBBWYJUICE	0.4 x	0.74			
	PROFIT	0,4×	0.54			
			0			

STEP 2: Constraints and Objective? STEP 3: Graph STEP 4: Corners STEP 5: Evaluate objective CONSTRAINTS @ TOTAL APPLE JUICE ≤ 12000 => 0.6×+0.3y ≤ 12000 @TOTAL CRAN, JUICE \$ 1000 \$ 0,4x+0.7y \$ 11000 AND 3×20 AND BY20 OBJECTIVE 12000 $\begin{array}{l} PROFIT = P(x,y) = 0.4 \times +0.5 y \\ \hline PROFIT = P(x,y) = 0.5 \times +0.5 y \\ \hline PROFIT = P(x,y) = 0.5 \times +0.5 \\ \hline P$



The feasible region for the juice example Potted lines corresponding to all points where profit is \$2,000, \$4,000, \$6,000, *8,000, and \$10,000 are drawn.



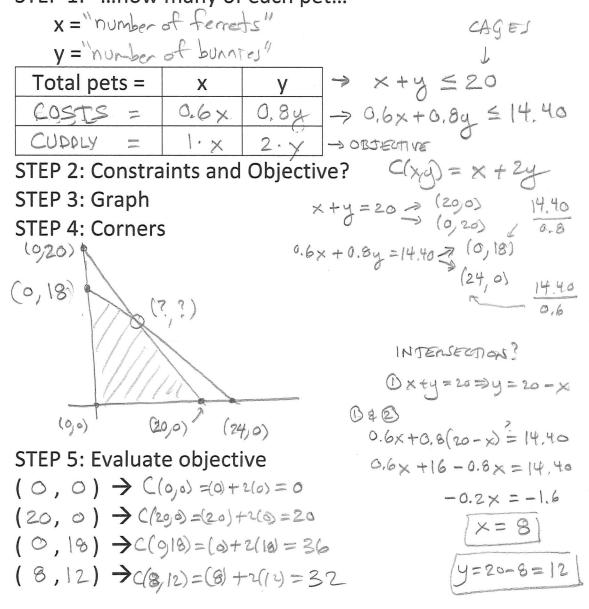
$$\frac{\text{COENERS}}{\text{PROFIT}} \xrightarrow{\text{OBJECTIVE}} \begin{array}{l} \text{PROFIT} = (0.4 \times + 0.5 \text{y} = P(x,y)) \\ (0,0) \rightarrow P(0,0) = 0.4(0) + 0.5(0) = 40 \\ \hline \\ 20000, 0) \rightarrow P(20000,0) = 0.4(20000) + 0.5(0) = 48000 \\ \hline \\ 20000, 0) \rightarrow P(20000,0) = 0.4(20000) + 0.5(0) = 48000 \\ \hline \\ 0, 15714.29) \rightarrow P(0,15714.24) = 0.4(0) + 0.5(11714.24) \\ = 47857.14 \\ \hline \\ 17000, 6000, 0 \rightarrow P(17000,6000) = 0.4(17000) + 0.5(6000) \\ \rightarrow = 49800 \\ \hline \\ \text{MAX} \quad PROFIT = 49800 \\ \hline \\ \text{MAX} \quad PROFIT = 49800 \\ \hline \\ 17000 \quad gallons \quad of \quad Apple - Crain \quad and \\ 6000 \quad gallons \quad of \quad Crain - Apple . \\ \end{array}$$

- ² Gina inherits a large sum of money and STEP 1: "...how many of each pet..."
- bunch of pet cages from an aunt. She decides to rescue some unwanted pets from a shelter.

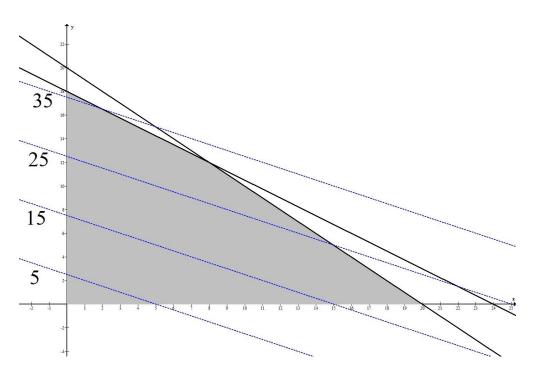
She has 20 cages that can each house either a bunny or a ferret.

On average it costs \$0.60 a day to feed one ferret and \$0.80 a day to feed one burny. Gina can budget no more than 14.40 a day for pet food. But cuddliness is an issue for Gina. She figures that bunnies are twice as cuddly as ferrets. That is, ferrets are each worth one cuddle-unit, while bunnies are each worth two.

Determine how many of each pet Gina could adopt to maximize cuddliness while staying within her budget and without buying more cages.



MAX CUDOLINESS = 36 WHICH OCCURS WHEN GINA GETS OFERNETS AND 18 BUNNIES. <u>The feasible region (Gina's pets)</u> Dotted lines correspond to all points where cuddliness is 5, 15, 25, and 35.



From Homework) Two foods contain
y proteins, carbohydrates, and fats.
Food A costs \$1 per pound and contains
30% protein and 50% carbohydrates.
Food B costs \$1.50 per pound and
contains 20% protein and 75%
carbohydrates.

What combination of these two foods provides at least 1 pound of protein, 2.5 pounds of carbohydrates, and 0.25 pownds of fat at the lowest cost?

STEP 1: "What ... these two foods..." X ="pounds of Food A" y ="pounds of Food B" Х amount = y 1.50. 4 COSTS = ·× 0.3× PROTEIN = 0.2.4 0.254 CARBS = 6.5× FAT 0.2× 0.054 and the second 0.3×+0.2y Z G. 5 x + 0.75 y 22.5 6.2×+0.05y 20.25

STEP 2: Constraints and Objective? STEP 3: Graph OBJECTIVE: C(X) = X + 1.5 y **STEP 4: Corners** 1) 0.3×+0.2y=1 3) (3.3,0) (2) $0.5 \times +0.75y = 2.5 \rightarrow (0, 3.3)$ (5,0)3 0.2×+0.05y=0.25 -> (0,3) (1.25,0) CORNERS? (0,5) 00.3×+0.2y=1 (20,5x+0.75y=2.5. (0,3.3) 00.2y=1-0.3× 4=5-1.8× (1.25) (3.5,0) (5,0)2) 0\$2 0.5x+0.75(5-1.5x)=2.5 **STEP 5: Evaluate objective** 0.5×+3.75-1.125×=2.5 $(5, 0) \rightarrow ((5, 0) = (5) + 1.5(0) = 45$ $(2, 2) \rightarrow C(2, 2) = (2) + 1.5(2) = 45$ -0.625 x = -1.25 $(0, 5) \rightarrow C(0, 5) = (0) + 1.5(5) = 7.5$ $y = S - 1.5(^{b}) = 2$ LOWEST COST = #5 IT occurs when (2,2) TEST X=2 & y=2 (0,0) DOES NOT X=5 \$ 4=0 WORN INTHESE (on ANY WHENE ON OTHIS EDGE) SHADE OTHER SIDE!

4. Your company makes two household cleaners: Miracle Bathtub Cleaner and Speedex Floor Cleaner. Your daily production of both cleaners combined is limited to 2,000 gallons. Your daily sales of Miracle Bathtub Cleaner never exceed 1,200 gallons, and your daily sales of Speedex Floor Cleaner never exceed 1,400 gallons. Finally, you make \$1.00 profit on each gallon of Miracle Bathtub cleaner that you sell and \$2.00 on each gallon of Speedex Floor Cleaner that you sell.

Defermine the amount of each cleaner yaw should produce in order to maximize pro fit.

STEP 1: "...amount of each cleaner..." x = " gallons of Mir. Bathtub Cleaner" y = "gallons of Speedex Floor Cleaner" TOTAL GALLONS = Х Y $\Rightarrow OUTETIVE$ P(xy) = x + 2y2-4 PROFIT = l·× STEP 2: Constraints and Objective? STEP 3: Graph STEP 4: Corners (0, 2000) ① ×+y ≤2000 ≤ (2000,0) (1200,0) (1200,1000) 0 x+y= 2000 ② × ≤ 1200 (0,1400) (1002,1400) 3 y=1400 y ≤1400 3 > x=600 2000-(0,0) 6 1400 0 D x = 2000 X=1200 2000 1200 STEP 5: Evaluate objective =) y= 800 $(0, 0) \rightarrow P(0, 0) = (0) + 2(0) = 0$ $(0, 1400) \rightarrow P(9, 1400) = (0) + 2(1400) = \frac{3}{2800}$ (1200, 0) →P(1200,0) = (1200)+2(0)= (1200) (600,1400) → P(600,1400) = (600) + 2(1400) = \$400 = mAx! $(1200, 800) \rightarrow P(1200, 800) = (1200) + 2(800) = 92800$ MAX PROFIT = \$3400 AND IT OCCURS WHEN YOU PRODUCE X = 600 gallous of miracle Body Cleaner AND y= 1400 gallons of Speedex Floor Cleaner

Preview of how we will set up the problems in 4.2

Example 1: (From Homework)

Newjet, Inc. manufactures inkjet printers and laser printers. The company has the capacity to make 490 printers per day, and it has 840 hours of labor per day available. It takes 1 hour to make an inkjet printer and 3 hours to make a laser printer. The profits are \$90 per inkjet printer and \$140 per laser printer. Find the maximum profit.

Answer Set up	(how we start):		х	У	
x = number of	inkjet, y = number of laser	Hours per item	1	3	
		Profits per item	90	140	
Totals: Constraints:	Total hours = $x + 3y$, Total pro Total number of printers ≤ 49 Total number of hours ≤ 840 ,	$0, \qquad \text{so } x + y \le 4$		of printers	5 = x + y.

Objective: Total profits = P(x, y) = 90x + 140y.

Example 2: (From Homework)

A company manufactures two types of electric hedge trimmers, one of which is cordless. The cord-type trimmer requires 4 hours to make, and the cordless model requires 10 hours. The company has only 2000 work hours to use in manufacturing each day, and the packaging department can package only 300 trimmers per day.

Answer Set up (how we start):

	х	У
Hours per item	4	10

Totals:	Total hours = 4x + 10y,	Total number of trimmers = x + y.
Constraints:	Total number of trimmers ≤ 30	0, so $x + y \le 300$.
	Total number of hours ≤ 2000,	so $4x + 10y \le 2000$.
Objective:	None Given.	

Example 3: (From an old exam)

Your company makes two kinds of soda: Regular and Diet. Your total daily production of soda is limited to 1000 gallons. Production requires 2 cup of sugar per gallon of Regular and 1/2 cup of sugar per gallon of Diet. Today, you are limited to 626 cups of sugar. The profit is \$1 per gallon of Regular soda and \$1.20 per gallon of Diet soda

Answer Set up (how we start):

x = gallons of Regular, y = gallons of Diet

	х	у
Sugar per item	2	0.5
Profit per item	1	1.20

Totals:	Total sugar = 2x + 0.5y, Total profits = x + 1.20y, Total gallons = x + y.		
Constraints:	Total sugar ≤ 626,	SO	2x + 0.5y ≤ 626.
	Total gallons ≤ 1000,	SO	x + y ≤ 1000.
Objective:	Total Profits = P(x, y) =	x + 1	20y

Example 4 (From an old exam):

Oscar sells boxes of toys and balloons for children's parties. The Standard Box contains 30 toys and 60 balloons. The DeLuxe Box contains 75 toys and 90 balloons. Oscar has just done a complete check of his inventory and has found that he has 20,250 toys and 30,900 balloons currently in stock. He earns a profit of \$7 for each Standard Box and \$8 for each DeLuxe Box. How many of each Box Oscar should sell in order to maximize profit?

Answer Set up (how we start):

x = boxes of Standard , y = boxes of DeLuxe

	х	У
Toys per item	30	75
Balloons per item	60	90
Profit per item	7	8

Totals:	Toys = 30x + 75y, Balloons = 60x + 90y, Profits = 7x + 8y.			
Constraints:	so 30x + 75y ≤ 20250.			
	Balloons ≤ 30900,	so $60x + 90y \le 30900$.		
Objective:	Total Profits = $P(x, y) = 7x + 8y$			

Sections 4.2 Review

The method of **linear programming** is a procedure we will use to optimize (\max/\min) an *objective* subject to *constraints*. Here is the short version:

- Step 1: Label the two quantities.
 - Example: "How many pounds of each ... " would mean x = amount of first (in pounds), y = amount of second (in pounds)
- Step 2: Collect information from the problem. Make a table of rates for x and y, write out formulas for total amounts, and list any restrictions.
- Step 3: Give Constraints and Objective. Write down the inequalities for the constraints and write down the function for the objective. Note that the last sentence typically tells you the objective. For example, "...to minimize cost?" means that the cost function, C(x, y), is the objective you are minimizing. You will only use the objective at the end of the problem.
- Step 4: Sketch the Feasible Region. Draw the line and shade the region that corresponds to each of the constraint inequalities. Indicate the overlapping region (this is the feasible region).
- Step 5: Find all the corners for the overlapping Feasible Region. You probably will need to do some intersections to do this. Use your picture to figure out which lines you need to intersect.
- Step 6: Plug each corner into the objective. The largest output is the maximum and the smallest output is the minimum.

We did four big examples in lecture. Here are a three more for you to try (full answers are on the following pages):

Lawn Care Example: A lawn care company has two types of fertilizer Regular and Deluxe. The profit for Regular is \$0.75 per bag and the profit for Deluxe is \$1.20 per bag. One bag of Regular has 3 pounds of active ingredient and 7 pounds of inert ingredients. One bag of Deluxe has 4 pounds of active ingredients and 6 pounds of inert ingredients. The warehouse has a limit of 8400 pounds of active ingredient and 14100 pounds on inert ingredient.

How many bags of each should we make to maximize profit?

A Boring No Words Question: Find the maximum of the objective f(x, y) = 14x + 20y subject to the constraints: $4x + 6y \le 1800$, $x \le 300$, $y \le 150$, $x \ge 0$, and $y \ge 0$.

Cookie Company Example: Your company makes two types of chocolate chip cookies. Each bag of 'Chocolate Lite' contains 10 ounces of chocolate chips and 35 ounces of dough. Each bag of 'Chocolate Overload' contains 20 ounces of chocolate chips and 25 ounces of dough. The profit on each bag of Chocolate Lite is \$1.10, while the profit on each bag of Chocolate Overload is \$0.80. Your company current has 1000 ounces of chocolate chips and 2555 ounces of cookie dough.

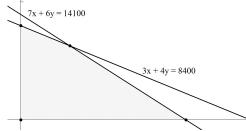
How many bags of each should you produce to maximize profit?

ANSWER to Lawn Care Example:

Let x = number of bags of Regular and y = number of bags of Deluxe. Here is the collected information:

		x	y	Total formula	Constraints/Objective
_	Active	3	4	3x + 4y	$3x + 4y \le 8400$
	Inert	7	6	7x + 6y	$7x + 6y \le 14100$
	Profit	0.75	1.20	0.75x + 1.20y	P(x,y) = 0.75x + 1.20y = Objective

Now we get points, draw the lines and shade the feasible region: 3x + 4y = 8400 goes through $(0, \frac{8400}{4}) = (0, 2100)$ and $(\frac{8000}{3}, 0) = (2800, 0)$. 7x + 6y = 14100 goes through $(0, \frac{14100}{6}) = (0, 2330)$ and $(\frac{141000}{7}, 0) \approx (2014.29, 0)$. Note that (0, 0) works in both of these constraints so we shade the overlapping 'origin-side' of the lines.



To get the corner at the intersection, we combine: (i) 3x + 4y = 8400 and (ii) 7x + 6y = 14100(i) $3x + 4y = 8400 \Rightarrow 4y = 8400 - 3x \Rightarrow y = 2100 - 0.75x$. (i) and (ii) $7x + 6(2100 - 0.75x) = 14100 \Rightarrow 7x + 12600 - 4.5x = 14100 \Rightarrow 2.5x = 1500 \Rightarrow x = \frac{1500}{2.5} \approx 600.$ And y = 2100 - 0.75x = 2100 - 0.75(600) = 1650

Evaluating the objective at the found corners gives:

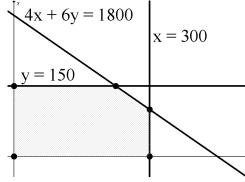
P(0,0)= 0.75(0) + 1.20(0)= 0 dollars P(0, 2100)= 0.75(0) + 1.20(2100)= 2520 dollars = 0.75(2014.29) + 1.20(0) = 1510.71 dollars P(2014.29, 0)P(600, 1650)= 0.75(600) + 1.20(1650) = 2430 dollars

Thus, the maximum profit is \$2520 and it occurs when you produce x = 0 bags of Regular and y = 2100 bags of Deluxe.

The problem is already set up for us, so we just need to graph and find corners. First get points, draw the lines and shade the feasible region:

4x + 6y = 1800 goes through $(0, \frac{1800}{6}) = (0, 300)$ and $(\frac{1800}{4}, 0) = (450, 0)$. y = 150 is a horizontal lines at y = 150.

x = 300 is a vertical line at x = 300. Note that (0,0) works in these three constraints so we shade the overlapping 'origin-side' of the lines (inside the first quadrant, that is what $x \ge 0$ and $y \ge 0$ is telling us).



To get the corners at the intersections, we combine.

One corner: (i) 4x + 6y = 1800 and (ii) y = 150

(i) and (ii) $4x + 6(150) = 1800 \Rightarrow 4x + 900 = 1800 \Rightarrow 4x = 900 \Rightarrow x = \frac{900}{4} \approx 225$. So one of the corners is (225, 150).

The other corner: (i) 4x + 6y = 1800 and (ii) x = 300

(i) and (ii) $4(300) + 6y = 1800 \implies 1200 + 6y = 1800 \implies 6y = 600 \implies y = \frac{600}{6} \approx 100$. So the other corner is (300, 100).

Evaluating the objective at the found corners gives:

f(0,0)	= 14(0) + 20(0)	= 0
f(0, 150)	= 14(0) + 20(150)	= 3000
f(300, 0)	= 14(300) + 20(0)	= 4200
f(300, 100)	= 14(300) + 20(100)	= 6200
f(225, 150)	= 14(225) + 20(150)	= 6150
it occurs who	a = 300 and $u = 100$	1

Thus, the maximum is 6200 and it occurs when x = 300 and y = 100.

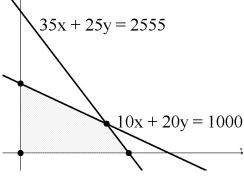
ANSWER to Cookie Company Example:

Let x = number of bags of Chocolate Lite and y = number of bags of Chocolate Overload. Here is the collected information: .

	x	y	Total formula	Constraints/Objective
Chips	10	20	10x + 20y	$10x + 20y \le 1000$
Dough	35	25	35x + 25y	$35x + 25y \le 2555$
Profit	1.10	0.80	1.10x + 0.80y	P(x,y) = 1.10x + 0.80y = Objective

Now we get points, draw the lines and shade the feasible region:

10x + 20y = 1000 goes through $(0, \frac{1000}{20}) = (0, 50)$ and $(\frac{1000}{10}, 0) = (100, 0)$. 35x + 25y = 2555 goes through $(0, \frac{2555}{25}) = (0, 102.2)$ and $(\frac{2555}{35}, 0) \approx (73, 0)$. Note that (0, 0) works in both of these constraints so we shade the overlapping 'origin-side' of the lines.



To get the corner at the intersection, we combine: (i) 10x + 20y = 1000 and (ii) 35x + 25y = 2555(i) $10x + 20y = 1000 \Rightarrow 20y = 1000 - 10x \Rightarrow y = 50 - 0.5x$. (i) and (ii) $35x + 25(50 - 0.5x) = 2555 \Rightarrow 35x + 1250 - 12.5x = 2555 \Rightarrow 22.5x = 1305 \Rightarrow x = \frac{1305}{22.5} \approx 58.$ And y = 50 - 0.5x = 50 - 0.5(58) = 21.

Evaluating the objective at the found corners gives:

$$\begin{array}{lll} P(0,0) &= 1.10(0) + 0.80(0) &= 0 \text{ dollars} \\ P(0,50) &= 1.10(0) + 0.80(50) &= 40 \text{ dollars} \\ P(73,0) &= 1.10(73) + 0.80(0) &= 80.30 \text{ dollars} \\ P(58,21) &= 1.10(58) + 0.80(21) &= 80.60 \text{ dollars} \end{array}$$

Thus, the maximum profit is \$80.60 and it occurs when you produce x = 58 bags of Chocolate Lite and y = 21bags of Chocolate Overload.

Section 4.1 / Problem 4: Example of Graphing Inequalities in WebAssign Graphing Tool

Section 4.1 / Problem 4: Graph the region given by the set of inequalities

(1) $x + 2y \le 48;$ (2) $x + y \le 30;$ (3) $2x + y \le 50;$ (4) $x \ge 0;$ (5) $y \ge 0$

STEP 1: Graph a line for EVERY inequality. In this example you will graph 5 lines. You need to find a couple points on each line that are in the graphing window. We will discuss in lecture how you can quickly find these....

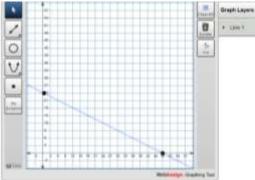
x+2y = 48 goes thru (0,24) and (48,0); x + y = 30 goes thru (0,30) and (30,0);

2x + y = 50 goes thru (0,50) and (25,0); x = 0 goes thru (0,0) and (0,10). y = 0 goes thru (0,0) and (5,0).

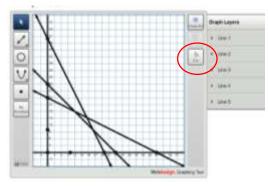
Plot each line with 3 clicks as follows:

- (a) 1 Click Click on the double-arrow line (do not click "ray" or "line segment").
- (b) 2 More Clicks Click on the two points.

Here I clicked on (0,24) and (48,0) and the line appeared...

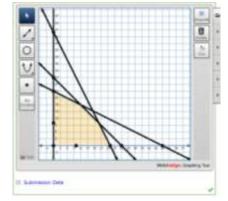


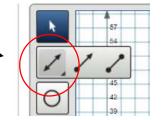
Here is what it looks like after I plotted all 5 lines...



STEP 2: Click "Fill" (shown circled above) and click any point in the region that satisfies all the inequalities.

In this case (1,1) works in all the inequalities you click any point the region containing (1,0). Here is the answer with a "Green Check" for correct, success!





NOTE: If you click on the link that says "Line 1" you can see and adjust the points you clicked, it looks like this... (please check this for each line)

